Question	Scheme	Marks	AOs
1(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$f(1.8) = 7 - 2\times1.8^2 = 0.52 \Rightarrow gf(1.8) = g(0.52) = \dfrac{3\times0.52}{5\times0.52-1} = ...$	M1	1.1b
	$gf(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(g^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
		(5 marks)	

Notes

(a)

B1: Correct range. Allow f(*x*) or f for *y*. Allow e.g. $\{y \in \mathbb{R}: y \leqslant 7\}, -\infty < y \leqslant 7, (-\infty, 7]$

(b)

M1: Full method to find f(1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute *x* = 1.8 into an attempt at gf(*x*).

E.g. $gf(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3\left(7-2(1.8)^2\right)}{5\left(7-2\times(1.8)^2\right)-1} = ...$

A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out *x* from an *xy* term and an *x* term.
If they swap *x* and *y* at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out *y* from an *xy* term and a *y* term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}, \dfrac{1}{5}+\dfrac{3}{25x-15}$

Ignore any domain if given.

Question	Scheme	Marks	AOs
2(a)	Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ Or substitutes $x=\frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3}=\right)4\pm\frac{...}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3}=\right)4-\frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0\leqslant g^{-1}(x)\leqslant 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0)=\frac{8\times0+5}{2\times0+3}$ **or** $f(4)=\frac{8\times4+5}{2\times4+3}$	M1	3.1a
	Attempts both boundaries $f(0)=\frac{8\times0+5}{2\times0+3}$ **and** $f(4)=\frac{8\times4+5}{2\times4+3}$	dM1	1.1b
	Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x)=\sqrt{16-x}\Rightarrow fg^{-1}(x)=\frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $fg^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **or** $fg^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	M1	3.1a
	$fg^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **and** $fg^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	dM1	1.1b
	Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$	A1	2.1
		(3)	
			(8 marks)
Notes:			

(a)

M1: Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ You can condone poor algebra as long as they reach a value for *x*.

Alternatively attempt to substitute $x=\frac{3}{2}$ into $f^{-1}(x)=\frac{\pm5\pm3x}{\pm2x\pm8}$ or equivalent (may be in terms of *y*). Note that attempts to find e.g. $f'(x)$ or $\frac{1}{f(x)}$ which may be implied by values such as $\frac{6}{17}, \frac{17}{6}, \frac{7}{18}, \frac{18}{7}$ score M0

A1: Achieves $\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$. Do not be concerned what they call it, just look for the value e.g. $x=-\frac{1}{10}$ or just $-\frac{1}{10}$ is fine. Correct answer with no (or minimal) working scores both marks.

(b)

M1: Attempts to divide $8x+5$ by $2x+3$

Look for $4\pm\frac{...}{2x+3}$ where … is a constant or $8x+5=A(2x+3)+B$ with A or B correct (which may be in a fraction) or in a long division attempt and obtains a quotient of 4 or attempts to express the numerator in terms of the denominator e.g. $\frac{8x+5}{2x+3}=\frac{4(2x+3)+...}{2x+3}$

A1: A full and complete method showing $\frac{8x+5}{2x+3}=4-\frac{7}{2x+3}$ or $\frac{8x+5}{2x+3}=4+\frac{-7}{2x+3}$

Also allow for correct values e.g. $A = 4$, $B = -7$

Do not isw here e.g. if they obtain $A = 4$, $B = -7$ and then write $-7+\frac{4}{2x+3}$ score A0

(c)

B1: Deduces $0\leqslant g^{-1}(x)\leqslant 4$ o.e.

E.g. $0\leqslant y\leqslant 4$, $0\leqslant \text{range}\leqslant 4$, $g^{-1}(x)\leqslant 4$ and $g^{-1}(x)\geqslant 0$, $0\leqslant g^{-1}\leqslant 4$, $[0, 4]$

but not e.g. $0\leqslant x\leqslant 4$, $0\leqslant g(x)\leqslant 4$, $(0, 4)$

(d)

M1: Attempts either boundary. Look for either $f(0)=\frac{8\times0+5}{2\times0+3}$ **or** $f(4)=\frac{8\times4+5}{2\times4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times0+3}$ **or** $f(4)=4-\frac{7}{2\times4+3}$

dM1: Attempts both boundaries. Look for $f(0)=\frac{8\times0+5}{2\times0+3}$ **and** $f(4)=\frac{8\times4+5}{2\times4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times0+3}$ **and** $f(4)=4-\frac{7}{2\times4+3}$

A1: Correct answer written in the correct form.

E.g. $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant \text{range}\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant y\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}$ and $fg^{-1}(x)\geqslant\frac{5}{3}$

$\frac{5}{3}\leqslant fg^{-1}\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}\cap fg^{-1}(x)\geqslant\frac{5}{3}$, $\left[\frac{5}{3}, \frac{37}{11}\right]$ but not e.g. $\frac{5}{3}\leqslant x\leqslant\frac{37}{11}$

PTO for an alternative to (d)

(d) **Alternative:**

M1: Attempts $fg^{-1}(x)$ and either boundary using $x = 0$ or $x = 16$

Look for either $fg^{-1}(0) = \dfrac{8\times g^{-1}(0)+5}{2\times g^{-1}(0)+3}$ **or** $fg^{-1}(16) = \dfrac{8\times g^{-1}(16)+5}{2\times g^{-1}(16)+3}$

Or uses (b) e.g. $fg^{-1}(0) = 4 - \dfrac{7}{2\times g^{-1}(0)+3}$ **or** $fg^{-1}(16) = 4 - \dfrac{7}{2\times g^{-1}(16)+3}$

The attempt at $fg^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

dM1: Attempts both boundaries. Look for $fg^{-1}(0) = \dfrac{8\times g^{-1}(0)+5}{2\times g^{-1}(0)+3}$ **and** $fg^{-1}(16) = \dfrac{8\times g^{-1}(16)+5}{2\times g^{-1}(16)+3}$

Or uses (b) e.g. $fg^{-1}(0) = 4 - \dfrac{7}{2\times g^{-1}(0)+3}$ **and** $fg^{-1}(16) = 4 - \dfrac{7}{2\times g^{-1}(16)+3}$

The attempt at $fg^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

A1: Correct answer written in the correct form with exact values.

E.g. $\dfrac{5}{3} \leqslant fg^{-1}(x) \leqslant \dfrac{37}{11}$, $\dfrac{5}{3} \leqslant \text{range} \leqslant \dfrac{37}{11}$, $\dfrac{5}{3} \leqslant y \leqslant \dfrac{37}{11}$, $fg^{-1}(x) \leqslant \dfrac{37}{11}$ and $fg^{-1}(x) \geqslant \dfrac{5}{3}$

$\dfrac{5}{3} \leqslant fg^{-1} \leqslant \dfrac{37}{11}$, $fg^{-1}(x) \leqslant \dfrac{37}{11} \cap fg^{-1}(x) \geqslant \dfrac{5}{3}$, $\left[\dfrac{5}{3}, \dfrac{37}{11}\right]$ but not e.g. $\dfrac{5}{3} \leqslant x \leqslant \dfrac{37}{11}$

Note that the $\dfrac{37}{11}$ is sometimes obtained fortuitously from incorrect working so check working carefully.

Question	Scheme	Marks	AOs
3(a)	$f(x) > 33$	B1	1.1b
		(1)	
(b)	$y = 3 + \sqrt{x-2} \Rightarrow x = ...$	M1	1.1b
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b
	$x > 33$	B1ft	2.2a
		(3)	
(c)	$f(6) = 3 + \sqrt{6-2} = 5 \Rightarrow g("5") = \dfrac{15}{"5"-3} = ...$	M1	1.1b
	$= \dfrac{15}{2}$	A1	1.1b
		(2)	
(d)	$3 + \sqrt{a^2 + 2 - 2} = \dfrac{15}{a-3} \Rightarrow "a^2 - 9 = 15"$	M1	1.1b
	$a = 2\sqrt{6}$	A1	2.2a
		(2)	
			(8 marks)

Notes

(a)

B1: $f(x) > 33$o.e.

e.g. $y > 3$, range > 3, $f(x) \in (3, \infty)$, $\{f(x) : f(x) > 3\}$, $f > 3$ **but not e.g.** $x > 3$, $f(x) \ldots 33$ $[3, \infty)$

(b)

M1: Sets $y = 3 + \sqrt{x-2}$ and attempts to make x the subject (or vice versa). Look for the correct order of operations so score for an expression of the form $(x =)\ (y \pm 3)^2 \pm 2$ or $(y =)\ (x \pm 3)^2 \pm 2$

A1: $f^{-1}(x) = (x-3)^2 + 2$ Also accept $f^{-1} : x \to (x-3)^2 + 2$. Condone $f^{-1} = (x-3)^2 + 2$ (or $f^{-1} = y = (x-3)^2 + 2$) but do not allow just $y = ...$ or $f^{-1} : y =$

Also accept other equivalent expressions such as $f^{-1}(x) = x^2 - 6x + 11$ (simplified or unsimplified)

B1ft: $x > 33$or follow through on their part (a). The omission of $x \in$ □ is condoned.

Allow equivalent answers such as $x \in ("3", \infty)$ or $\{x : x > "3"\}$

Note: It is also acceptable to define f^{-1} in any variable e.g. as $f^{-1}(t) = (t-3)^2 + 2$ $t > 33$as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $f^{-1}(t) = ...$ not just $f^{-1} = ...$

(c)

M1: Substitutes $x = 6$ into f and substitutes the result into g to find a value for $gf(6)$.

Allow an attempt to substitute $x = 6$ into $gf(x) = \dfrac{15}{\sqrt{x-2}}$ condoning slips. They must proceed to find a value. Condone arithmetical slips and bracket errors/omissions. Condone for M1 attempts where when dealing with $\sqrt{x-2}$ leads to two different answers e.g. $\dfrac{15}{\sqrt{6-2}} \to \pm\dfrac{15}{2}$

A1: $\dfrac{15}{2}$ only oe isw once a correct answer is seen

(d)

M1: Attempts to form the equation $3+\sqrt{a^2+2-2}=\frac{15}{a-3}$, and proceeds to a quadratic in *a* (usually $a^2=k$ or $a^2-k=15$ but condone arithmetical, miscopying and sign slips. Condone equations which would lead to complex roots.
May be implied by a correct exact answer.

Alternatively, they attempt to form the equation $a^2+2=\mathrm{f}^{-1}\mathrm{g}(a)\Rightarrow a^2+2=\left(\frac{15}{a-3}-3\right)^2+2$

$\Rightarrow(a+3)(a-3)=15\Rightarrow a^2-9=15$ (condone slips)

They should be square rooting both sides so that $\sqrt{a^2+2-2}\to a$, before multiplying both sides by $a-3$ and rearranging so that the a^2 term comes from their "$(a+3)(a-3)$"

May be implied by a correct exact answer for their quadratic in *a* but a correct decimal answer does not imply this mark.

A1: $(a=)\ 2\sqrt{6}$ or accept $\sqrt{24}$ (they must reject the negative solution if found as $\mathrm{f}\left(a^2+2\right)\neq \mathrm{g}(a)$ when $a=-2\sqrt{6}$) $\sqrt{6}\times\sqrt{4}$ is A0
isw $\sqrt{24}$ followed by $4\sqrt{6}$ (incorrect manipulation of the surd) but not followed by $\pm\sqrt{24}$ o.e.
A decimal answer on its own or multiple answers e.g. $\pm\sqrt{24}$ score A0.